J. Fluid Mech. (1970), *vol.* 42, *part* 2, *pp.* 249–255
Printed in Great Britain

High Reynolds number flow in a moving corner

By RODNEY J. TABACZYNSKI, DAVID P. HOULT AND JAMES C. KECK
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 2 May 1969 and in revised form 31 December 1969)

The problem of a piston moving in a cylinder is studied experimentally using flow visualization techniques. A vortex motion is observed at the piston face and cylinder wall interface as the cylinder wall moves toward the piston. Non-dimensional scaling parameters for the vortex size and stability are determined and semi-empirical theories for the size of the vortex are presented.

Introduction

When one plane moving in the direction of its normal slides over another plane which is perpendicular to it, a fluid motion is created in the corner between the two planes. This situation is observed whenever a piston moves in a cylinder or a blade scrapes fluid from a surface. Batchelor (1967, pp. 224–227) has treated this problem for the case of negligible inertial effects. However, for the practical problems of a water pump or a piston moving in the cylinder of an internal combustion engine, inertial effects are dominant when $r \gg \nu/U_w = 0{\cdot}001$ cm, where r is the distance from the piston-cylinder wall interface. Therefore, the greater portion of the flow field in the aforementioned problems is inertia dominated.

To simulate such flows an experiment was designed with a piston cylinder geometry and with water as the working fluid. In this investigation two types of flows were observed: a sink flow as the cylinder wall retreated from the piston and a spiral vortex flow as the cylinder wall moved toward the piston (see figure 1). In the present work only the latter will be investigated to determine the relevant parameters governing the size and stability of the vortex motion. Other vortical flow problems involving the roll up of cast-off vortex sheets have been studied extensively. Alexander (1965) presents a unified treatment of these problems and an extensive bibliography.

An experimental apparatus and techniques

A piston-cylinder geometry was chosen to eliminate surface and end effects and to facilitate construction of the apparatus. Figure 2 shows a schematic diagram of the test section where the pistons were fixed and the cylinder wall allowed to move with the desired velocity. The fixed piston reference frame served two purposes. First, it permitted the movement of the cylinder wall which had less inertia than the working fluid. Secondly, it allowed the camera to remain fixed which facilitated the photography. It was necessary to build a square tank filled with water about the cylinder wall so that the flow could be viewed without distortion. The test section was built entirely out of plexiglass, and the wall velocity was controlled by a constant speed motor. The four wall velocity profiles shown in figure 3 were investigated where a positive wall velocity denotes movement toward the piston. Only the velocity profiles denoted by 1, 3 (for $t < \frac{1}{2}T$), and 4 (for $\frac{1}{2}T < t < T$) will be discussed; the other profiles result in sink-type flows.

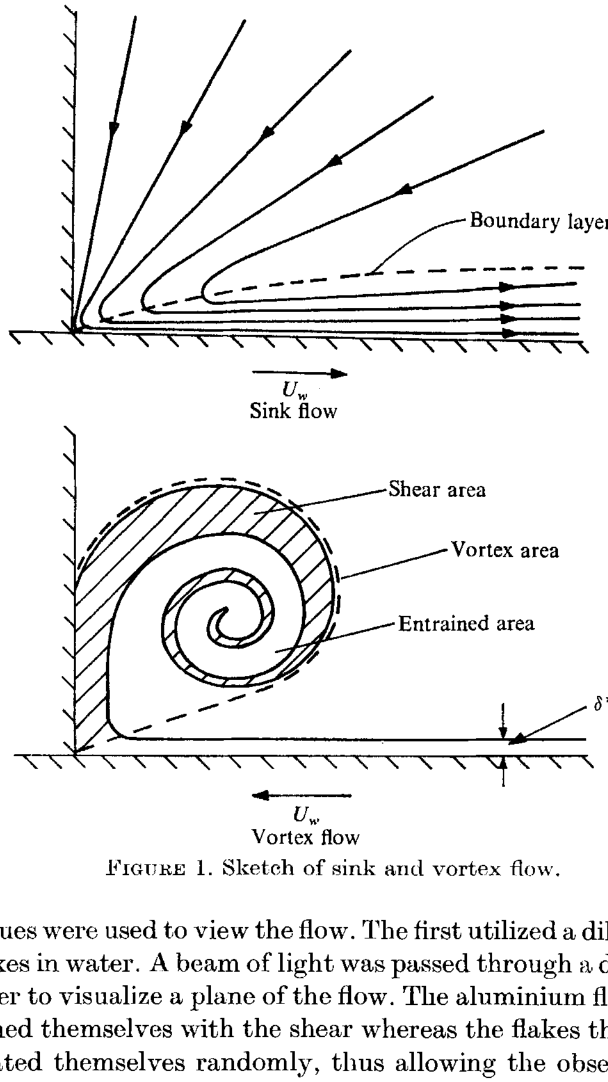
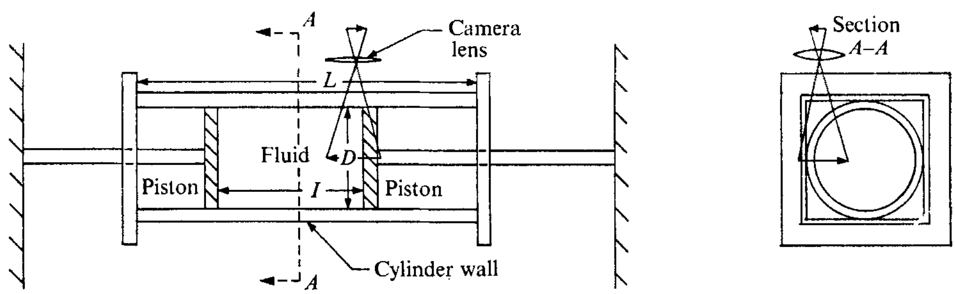


FIGURE 1. Sketch of sink and vortex flow.

Two techniques were used to view the flow. The first utilized a dilute solution of aluminium flakes in water. A beam of light was passed through a diameter of the cylinder in order to visualize a plane of the flow. The aluminium flakes that were in motion aligned themselves with the shear whereas the flakes that were not in motion orientated themselves randomly, thus allowing the observation of the motion in this plane. The second technique consisted of placing a strip of dye along the bottom of the cylinder and simply watching the motion of the dye as the wall moved. A movie camera was used to record the motion in both of these cases. Figure 4 (plate 1) shows typical flows using these techniques. Measurements of the vortex area and shear area are accurate to 25 %, and measurements of the

Reynolds numbers are accurate to 10 %. The principal sources of error are in determining the bounding area of the vortex and in determining the exact position of the cylinder wall as a function of time.



$I = 14$ cm, $L = 33$ cm, $D = 9{\cdot}5$ cm

FIGURE 2. Schematic of test section.

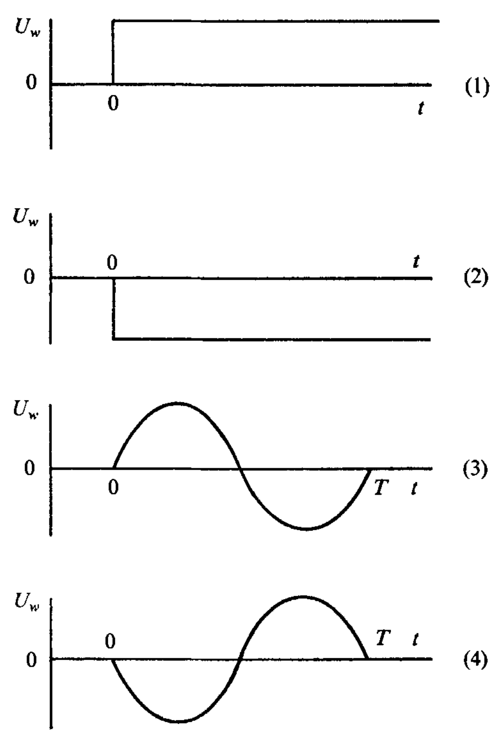


FIGURE 3. Velocity profiles.

Semi-empirical theories

It was observed that the characteristic size of the vortex was small compared with the diameter of the cylinder, the distance between the pistons, and the length of stroke. Hence, the vortex area can be expressed as

$$A = f(U_w, X, t, T, \mu, \rho),$$

where U_w is the wall velocity, X is the length of stroke, t is the time, T is the period, μ is the viscosity of the working fluid, and ρ is the fluid density. Forming the relevant dimensionless groups A/X^2 can be expressed as

$$\frac{A}{X^2} = f\left(\frac{U_w X}{\nu}, \frac{t}{T}, \frac{\sqrt{(\nu t)}}{U_w T}\right). \tag{1}$$

The non-dimensional group $\sqrt{(\nu t)}/U_w T$ is the ratio of the pressure gradient produced by an accelerating wall ($\partial p/\partial x$ unsteady $\approx \rho U_w/T$) to the pressure gradient produced by flow turning the corner ($\partial p/\partial x$ corner $= \rho U_w^2/\sqrt{(\nu t)}$, where $\sqrt{(\nu t)}$ is assumed to be the radius of curvature for the flow in the corner). In this work $\sqrt{(\nu t)}/U_w T \approx 0{\cdot}03 \ll 1$, indicating that for flow in the corner, moving the cylinder wall is equivalent to moving the piston for both steady and unsteady wall velocities. If the difference between the boundary-layer structure with an unsteady wall velocity and that with a steady wall velocity is not very important compared to the flow in the corner, then in determining the gross features of the flow, e.g. A/X^2, the flow should have a quasi-steady character; therefore,

$$\frac{A}{X^2} = f\left(\frac{U_w X}{\nu}\right).$$

For a constant wall velocity the vortex area in the stable flow régime was assumed proportional to the boundary-layer area A_b scraped up in a distance X, where

$$A_b = \int_0^x \delta^* \, dx = \frac{4}{3}\left(\frac{\nu X^3}{\pi U_w}\right)^{\frac{1}{2}} \quad \text{and} \quad \delta^* = \int_0^\infty \frac{U}{U_w} dy, \tag{2}$$

where δ^* is the displacement thickness for the problem of an impulsively started flat plate. Hence,

$$\frac{A}{X^2} = C\frac{A_b}{X^2} = C\frac{4}{3\pi^{\frac{1}{2}}}\left(\frac{U_w X}{\nu}\right)^{-\frac{1}{2}}, \tag{3}$$

where C is a constant to be determined from experiments. In the turbulent vortex flow régime, an elementary entrainment theory was proposed, where the rate of change of the vortex area was assumed proportional to the product of the exposed perimeter of the vortex and the velocity difference between the vortex flow and the stationary fluid. The velocity difference is approximated by the wall velocity, and the exposed perimeter is taken as $\sqrt{(\pi A)}$. The equation governing the rate of growth of the vortex area is given by

$$\frac{dA}{dt} = \alpha U_w \sqrt{(\pi A)},$$

where α is the entrainment parameter determined empirically. Integrating this equation, the expression for the area becomes

$$A = \tfrac{1}{4}\alpha^2\pi\left(\int_0^t U_w dt\right)^2 = \tfrac{1}{4}\alpha^2\pi X^2, \tag{4}$$

and, therefore, A/X^2 is constant.

Discussion of results

It was observed that stable, transitional, and unstable flows (figure 4, plate 1) were present. Figure 5 shows this transition from stable to unstable vortex flows is a function of $U_w X/\nu$, and indicates that, below values of $U_w X/\nu = 12{\cdot}5 \times 10^3$, the vortex flow is stable, and, above $U_w X/\nu = 17{\cdot}5 \times 10^3$, the vortex flow is fully turbulent. The parameter U_w in figure 5 was used simply to spread out the data thus facilitating evaluation of critical Reynolds numbers. For the stable flow régime, figure 6 shows that the area of shear layer A_s that forms the vortex is essentially the area of the boundary layer A_b scraped up in a distance X. Figure 6

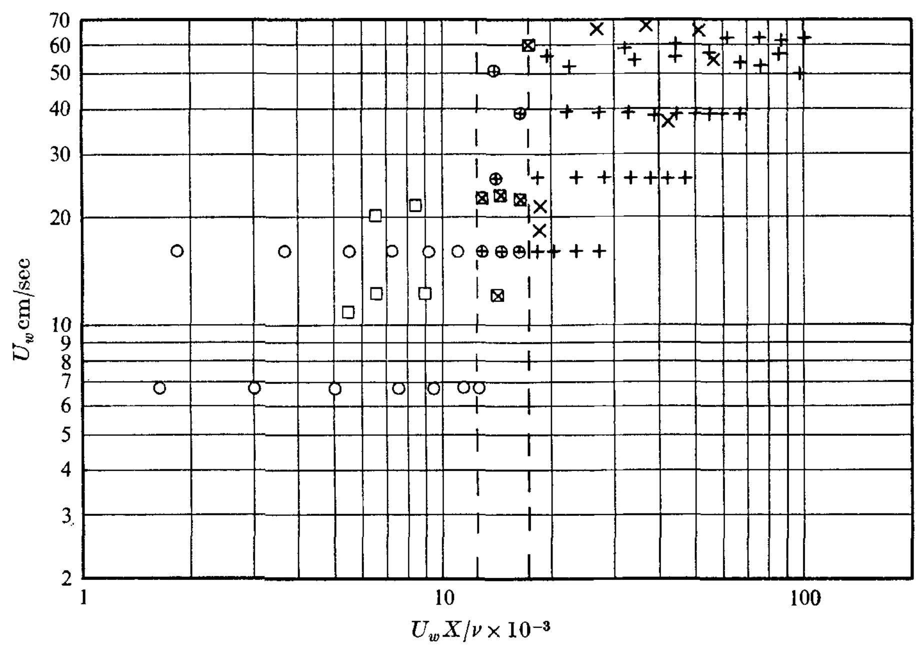


FIGURE 5. Flow character *vs.* Reynolds number.

U_w – const.	$Um \sin \omega t$	
○	□	Stable
⊕	⊠	Transitional
+	×	Turbulent

also shows that the vortex area A is proportional to the boundary-layer area A_b. The experimental data for the vortex area is correlated when it is non-dimensionalized with respect to the square of the stroke and plotted as a function of the local Reynolds number $U_x X/\nu$ as is shown in figures 7 and 8 for the constant velocity profiles and sinusoidal velocity profiles, respectively. From figures 6 and 7 the constant C defined by (3) is determined and has a value of 1·35. Equation (3) is plotted on figures 7 and 8 and is found to correlate the data in the stable régime within experimental error. Hence, the flow in this region can be considered as quasi-steady. Equation (4) is plotted on figures 7 and 8, and correlates the data for the constant velocity case when $\alpha = 0{\cdot}10$. However, the data for the sinusoidal

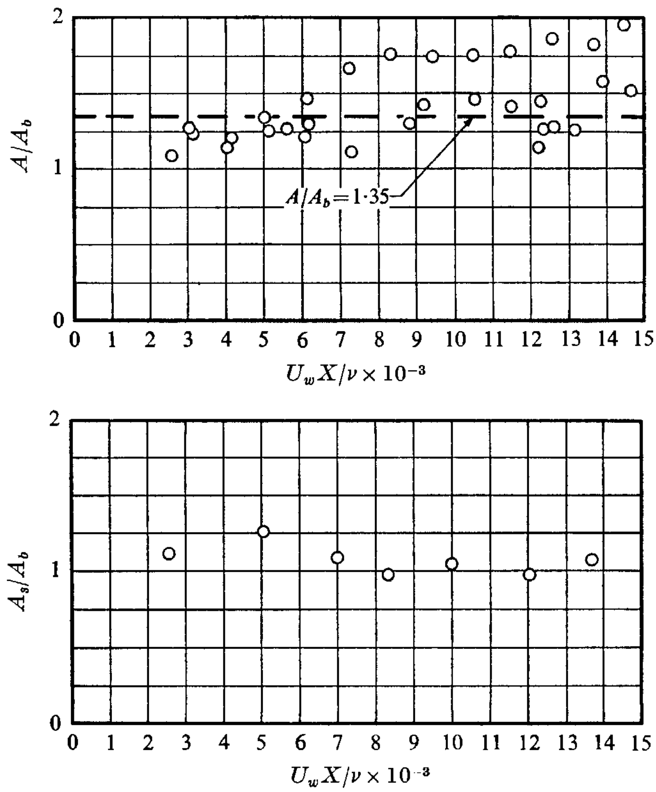


FIGURE 6. Ratios of vortex area and shear area to boundary-layer area.

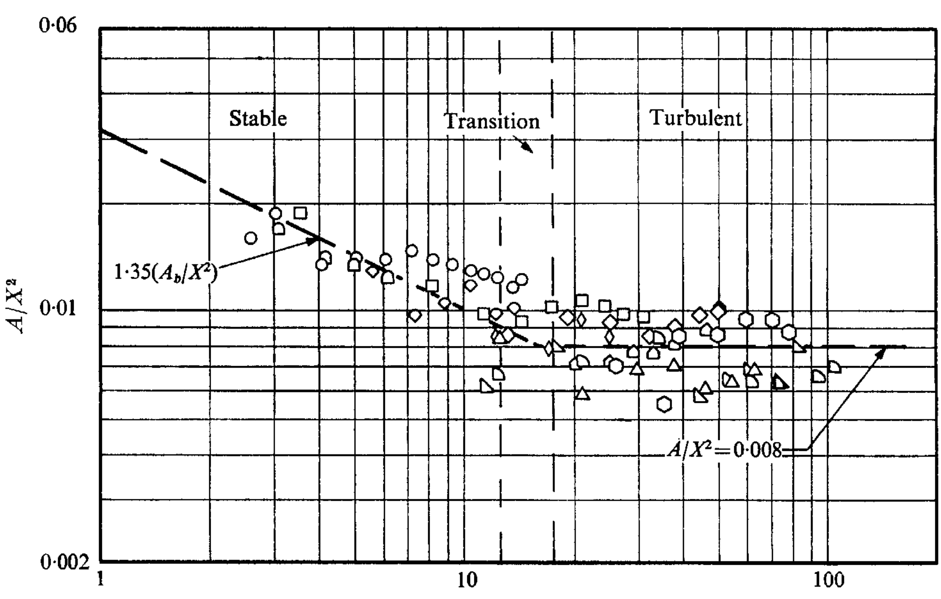


FIGURE 7. Ratio of vortex area to stroke squared *vs.* Reynolds number for constant wall velocities. U_w cm/sec:

7·1, 7·1, 15·7, 16·0, 25·6, 30·1, 34·0, 35·0, 50·1, 52·3, 58·3 case lie approximately 25 % lower than that of the constant velocity case. This is due to the more complicated flow field produced by using an unsteady wall velocity profile which leads to the breakdown of the assumption that the velocity difference across the shear layer can be taken as constant at an instant of time. A/X^2 can still be considered constant, and the entrainment parameter α has the value 0·087 as determined from figure 8. It should be noted that all the experimental data can be correlated with a 50 % accuracy if the intermediate value of $\alpha = 0{\cdot}096$ is used.

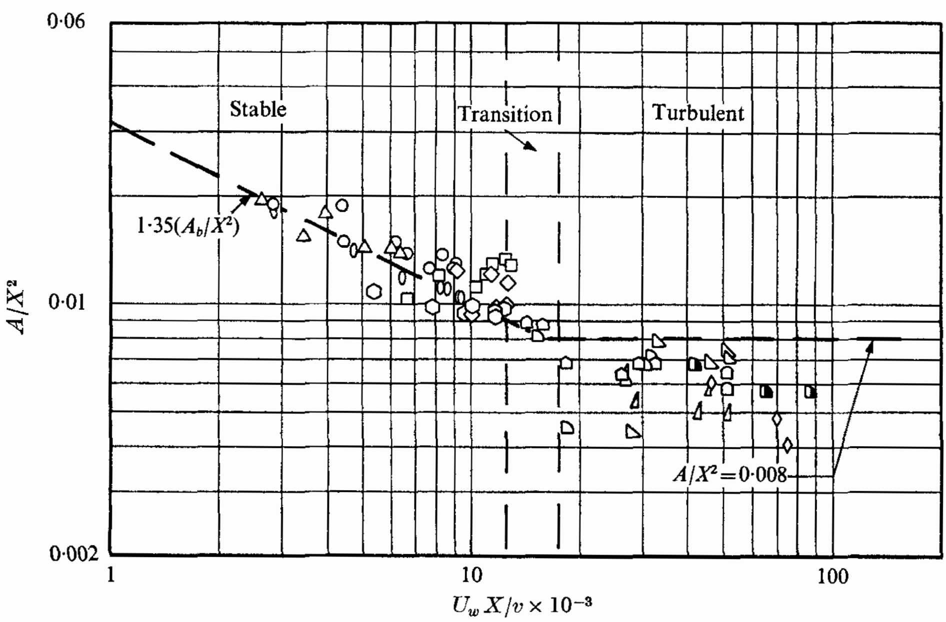


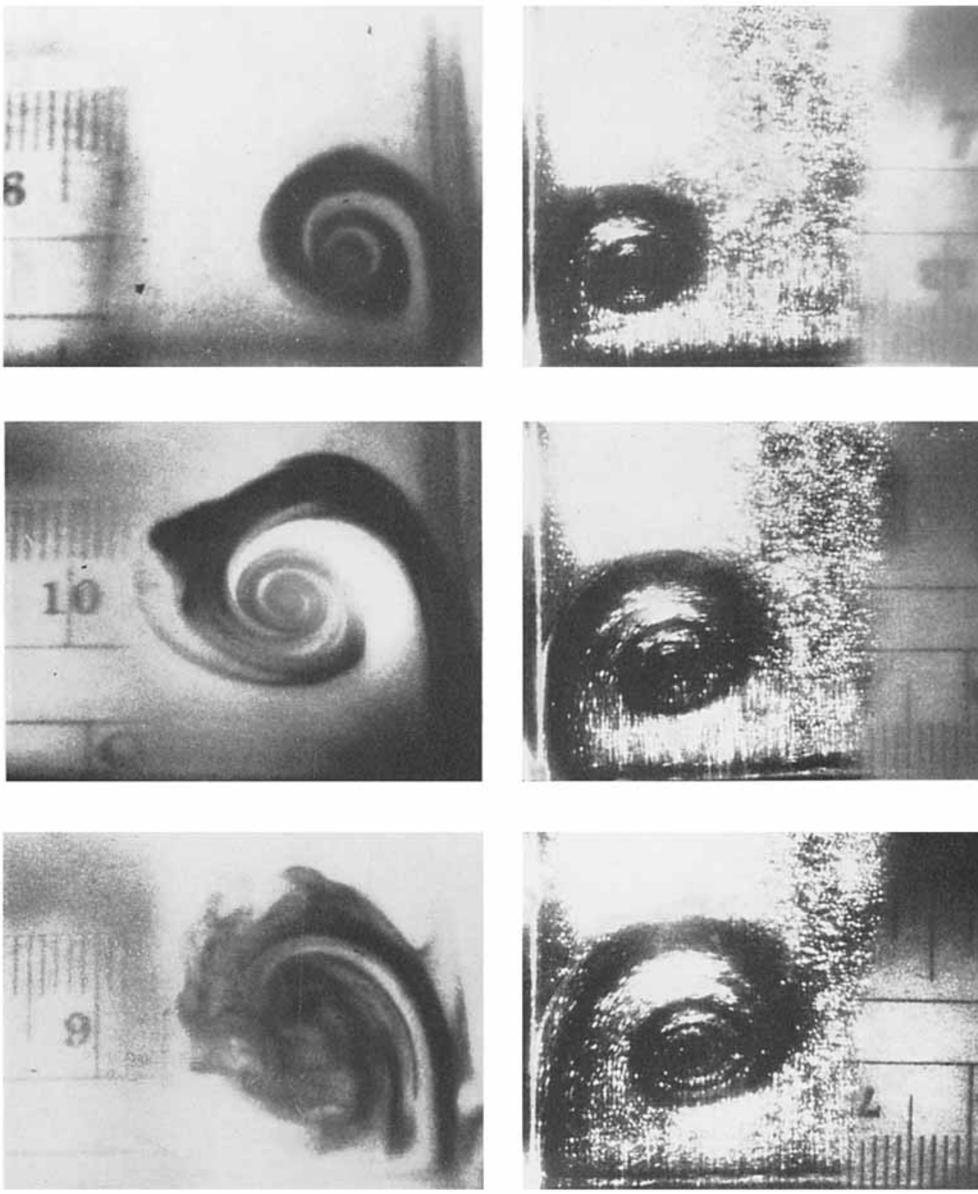
FIGURE 8. Ratio of vortex area to stroke squared *vs.* Reynolds number for sinusoidal wall velocities. U_w cm/sec:

○ $10{\cdot}5 \sin 2{\cdot}0t$	△ $7{\cdot}6 \sin 1{\cdot}4t$	$103{\cdot}0 \sin 19{\cdot}1t$	$-62{\cdot}5 \sin 11{\cdot}4t$
□ $15{\cdot}7 \sin 2{\cdot}9t$	$67{\cdot}8 \sin 12{\cdot}2t$	0 $-11{\cdot}7 \sin 2{\cdot}1t$	$-51{\cdot}1 \sin 12{\cdot}1t$
◇ $16{\cdot}8 \sin 3{\cdot}4t$	$45{\cdot}5 \sin 10{\cdot}8t$	⬡ $-15{\cdot}2 \sin 2{\cdot}8t$	$-96{\cdot}1 \sin 17{\cdot}6t$

REFERENCES

ALEXANDER, C. 1965 Self-similar hydrodynamics with vortex sheets. *AFOSR Scientific Report* 65–0642.

BATCHELOR, G. K. 1967 *An Introduction to Fluid Dynamics*. Cambridge University Press.



(*a*) (*b*)

FIGURE 4. (*a*) Stable, transitional, and turbulent vortex flow. (*b*) Growth of vortex for a constant velocity.